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Efficient Generic Search with Effect Handlers

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Programming Language Interest Group

(Joint work with Sam Lindley and John Longley)

A new complexity result for control operators

The crux of this work is to establish a new complexity result for control operators

Lay person's version of the result

There is a class of problems for which a language with control operators provides asymptotically more efficient solutions than a language without control operators ($\mathcal{O}(2^n)$ vs $\Omega(n2^n)$).

To establish the existence of this class, we use *generic search* as an example program and effect handlers as our control operator.

This talk is high-level walk-through of how we establish this result

(The possibility of the existence of this result can be traced back to Longley (2009))

(Disclaimer: we present the result using a contextual operational semantics, although, it was originally established using an abstract machine (Hillerström and Lindley 2016))

The plan of attack

- Define a pure functional language \mathcal{L} , and an extension thereof \mathcal{L}_{eff} with effect handlers.
- Provide a specification (type signature) of *generic search* problem
- Implement an efficient version of generic search in \mathcal{L}_{eff}
- ... and prove that it is indeed efficient
- Finally show that *any* implementation of generic search in \mathcal{L} has worse complexity

There is a single rule of engagement:

No change of types is allowed! (Longley and Normann 2015)

This rules out tricks such as

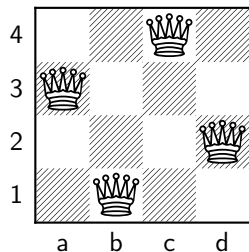
- CPS conversion (Hillerström et al. 2017)
- Implementing an interpreter for \mathcal{L}_{eff} in \mathcal{L}

Generic search

Given a search problem P , a generic search algorithm finds solutions of P .

Applications include (Daniels 2016)

- n -Queens
- Sudoku
- Finding Nash equilibria
- Graph-colouring
- Exact real number integration



A solution to 4-Queens problem

Rather than finding solutions of P , we *count* the number of solutions of P

First instance of efficient generic search with effect handlers

Somewhat related is work on exhaustive search on infinite spaces

- Berger (1990): exhaustive search on the Cantor space $2^{\mathbb{N}}$
- Escardó (2007): characterisation of searchable infinite sets
- Bauer (2011): efficient search on infinite sets with effect handlers

Fine-grain call-by-value PCF (Levy et al. 2003)

The core of a “pure” functional programming language \mathcal{L}

Types $A, B, C, D ::= \langle \rangle \mid \text{Bool} \mid \text{Nat} \mid A \times B \mid A + B \mid A \rightarrow B$

Values $V, W \in \text{Val} ::= x \mid b \in \mathbb{B} \mid n \in \mathbb{N} \mid \text{Plus} \mid \langle \rangle \mid \langle V; W \rangle$
 $\mid (\text{inl } V)^B \mid (\text{inr } W)^A \mid \lambda x^A. M \mid \text{rec } f^A x.M$

Computations $M, N \in \text{Comp} ::= V W$
 $\mid \text{let } \langle x; y \rangle = V \text{ in } N$
 $\mid \text{if } V \text{ then } M \text{ else } N$
 $\mid \text{case } V \{ \text{inl } x \mapsto M; \text{inr } y \mapsto N \}$
 $\mid \text{return } V$
 $\mid \text{let } x \leftarrow M \text{ in } N$

Eval. contexts $\mathcal{E} \in \text{Eval} ::= [] \mid \text{let } x \leftarrow \mathcal{E} \text{ in } N$

The static and dynamic semantics are completely standard.

Service announcement: Syntactic sugar

I shall permit myself to use regular call-by-value syntax, e.g. for $f, g, h, a \in \text{Val}$

$$f(h\ a) + g\langle \rangle$$

Service announcement: Syntactic sugar

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$$\llbracket f(h a) + g \langle \rangle \rrbracket = \mathbf{let} \ x \leftarrow h a \ \mathbf{in}$$
$$\mathbf{let} \ y \leftarrow f x \ \mathbf{in}$$
$$\mathbf{let} \ z \leftarrow g \langle \rangle \ \mathbf{in}$$
$$\text{Plus} \langle y; z \rangle$$

FGCB PCF with effect handlers

The language \mathcal{L}_{eff}

Handler types

$$F ::= C \Rightarrow D$$

Signatures

$$\Sigma ::= \cdot \mid \{\ell : A \rightarrow B\} \uplus \Sigma$$

Labels

$$\ell \in \mathcal{L}$$

Computations $M, N \in \text{Comp} ::= \dots \mid \mathbf{do} \ell V \mid \mathbf{handle} M \mathbf{with} H$

Handlers

$$H ::= \{\mathbf{val} x \mapsto M\} \mid \{\ell p r \mapsto N\} \uplus H$$

Eval. contexts

$$\mathcal{E} \in \text{Eval} ::= \dots \mid \mathbf{handle} \mathcal{E} \mathbf{with} H$$

FGCB PCF with effect handlers (dynamic semantics)

S-Ret **handle (return V) with H**

$\rightsquigarrow N[V/x],$

where $H^{\text{val}} = \{\mathbf{val} \ x \mapsto N\}$

S-Op **handle $\mathcal{E}[\mathbf{do} \ \ell \ V]$ with H**

$\rightsquigarrow N[V/p, \lambda y. \mathbf{handle} \ \mathcal{E}[\mathbf{return} \ y] \ \mathbf{with} \ H/r],$ where $H^\ell = \{\ell \ p \ r \mapsto N\}$

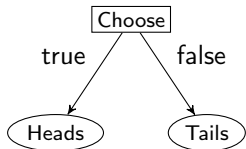
Example: Coin tossing (nondeterminism)

Fix $\Sigma = \{\text{Choose} : \text{Bool}\}$

A coin toss model

```
toss : ⟨ ⟩ → Toss  
toss = if do Choose then Heads  
      else Tails
```

Computation tree



A possible handler for Choose

```
allChoices : (⟨ ⟩ → Toss) → [Toss]  
allChoices = λm. handle m ⟨ ⟩ with  
          val x ↦ [x]  
          Choose r ↦ r true ++ r false
```

Enumerating all possible outcomes

$allChoices\ toss \rightsquigarrow^+ ??$

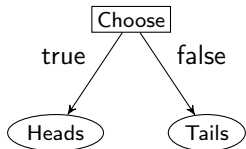
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Enumerating all possible outcomes

```
allChoices toss  $\rightsquigarrow^+$  [Heads, Tails]
```

Setting up generic search

The secret of generic search is *higher-order* functions

$$\text{Predicate} \doteq (\text{Nat} \rightarrow \text{Bool}) \rightarrow \text{Bool}$$

Setting up generic search

The secret of generic search is *higher-order* functions

$$\text{Point} \doteq \text{Nat} \rightarrow \text{Bool}$$
$$\text{Predicate} \doteq \text{Point} \rightarrow \text{Bool}$$

Setting up generic search

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Point \doteq Nat \rightarrow Bool

Predicate \doteq Point \rightarrow Bool

Counter \doteq Predicate \rightarrow Nat

Setting up generic search

The secret of generic search is *higher-order* functions

Point \doteq Nat \rightarrow Bool
Predicate \doteq Point \rightarrow Bool
Counter \doteq Predicate \rightarrow Nat

Some (silly) example predicates

$tt_n \doteq \lambda p.p\ 0; \dots ; p\ (n - 1); \mathbf{return\ true}$
 $div_n \doteq \mathbf{rec\ } d\ p.\mathbf{if\ } p\ (n - 1)\ \mathbf{then\ } d\ p\ \mathbf{else\ return\ } \mathbf{false}$
 $odd_n \doteq \lambda p.reduce\ xor\ \mathbf{false}\ [p\ 0, \dots, p\ (n - 1)]$

A pure generic search procedure

A possible implementation of generic search in \mathcal{L}

$count_n : (\text{Predicate} \rightarrow \text{Bool}) \rightarrow \text{Nat}$

$count_n \doteq \lambda pred. count' n (\lambda i. \perp)$

where

$count' 0 \quad p \doteq \text{if } pred\ p \text{ then } 1 \text{ else } 0$

$count' (n + 1) \quad p \doteq \quad count' n (\lambda i. \text{if } i = n \text{ then true else } p\ i)$
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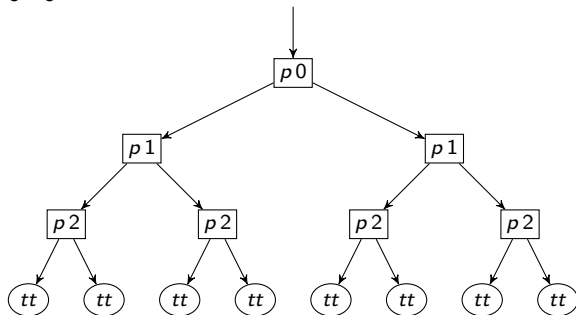
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Example $count_3 \ tt_3$:



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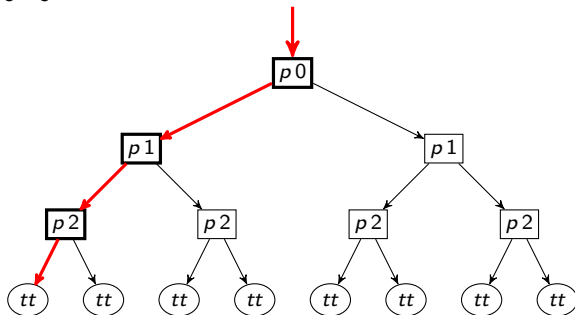
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Example $count_3 \ tt_3$: reaches the first leaf



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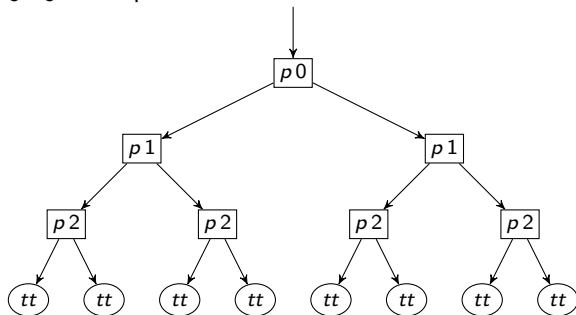
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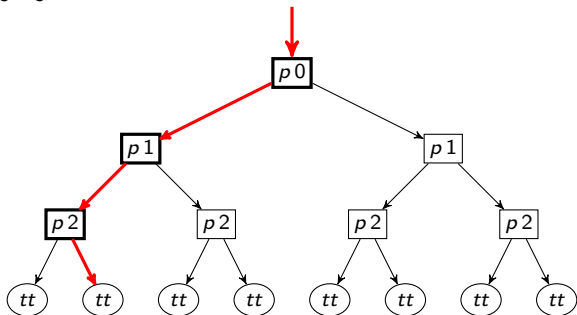
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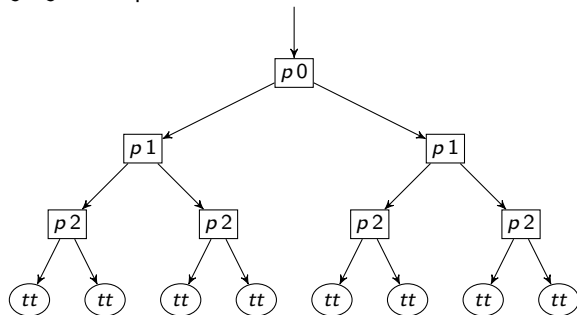
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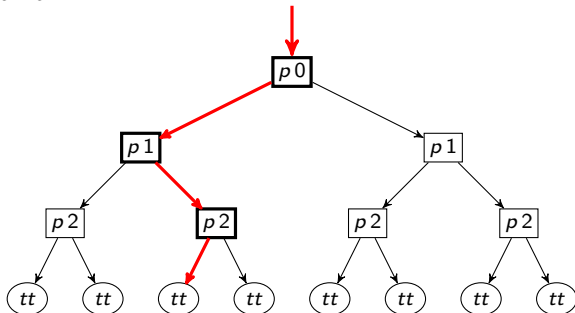
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Example $count_3 tt_3$: reaches the third leaf, etc. . .



The effectful generic search procedure

For the efficient implementation of generic search in \mathcal{L}_{eff} , we require one operation; fix $\Sigma \doteq \{\text{Branch} : \text{Bool}\}$

`genericSearch` : (Predicate \rightarrow Bool) \rightarrow Nat

`genericSearch` \doteq $\lambda pred.$ **handle** (if *pred* ($\lambda n.$ **do** Branch) **then** 1 **else** 0) **with**
 val $x \mapsto x$
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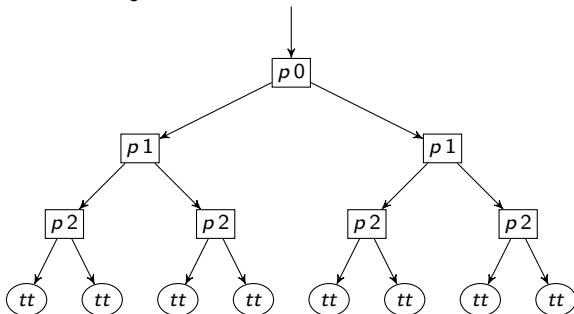
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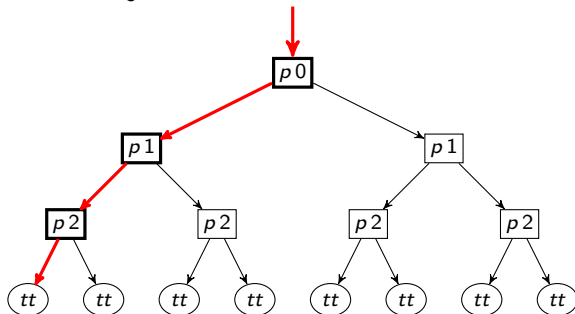
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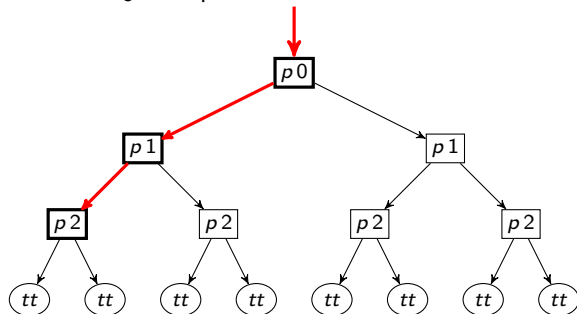
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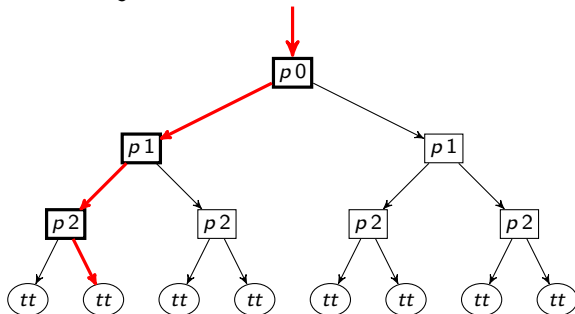
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Example `genericSearch tt3`: reaches the second leaf



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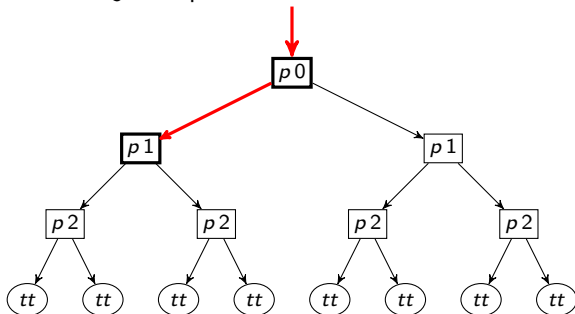
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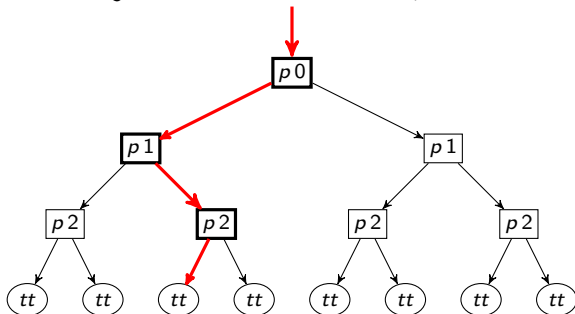
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Example $\text{genericSearch} \ tt_3$: reaches the third leaf, etc. . .



Semantics for predicates

Definition (The label set)

The set Lab consists of queries parameterised by a natural number and answers parameterised by a boolean, i.e. $\text{Lab} \doteq \{!tt, !ff\} \cup \{?n \mid n \in \mathbb{N}\}$

Definition (Decision tree)

A decision tree is a partial function $t : \mathbb{B}^* \rightarrow \text{Lab} \times \text{Eval} \times \mathbb{N}$ from lists of booleans to node labels with the following properties:

- The domain of t , $\text{dom}(t)$, is prefix closed.
- For any boolean, $b \in \mathbb{B}$, and list, $bs \in \mathbb{B}^*$, of booleans, if $t_\ell(bs) = !b$ is an answer node then bs is a leaf of t .

Notation: write t_ℓ and t_s for the projection of the first and third components of $t(-)$, respectively.

Definition

We implement the decision tree semantics as a partial function parameterised by an abstract point p , $\mathcal{T}_p : \text{Comp} \rightarrow (\mathbb{B}^* \rightarrow \text{Lab} \times \text{Eval} \times \mathbb{N})$, that given a predicate, $pred$, constructs a function, that given a list of booleans, bs , returns the corresponding node label in model of $pred$ p , where p is an “abstract point”.

$$\mathcal{T}_p(\mathbf{return\ true}) [] = (!\text{true}, [], 0)$$

$$\mathcal{T}_p(\mathbf{return\ false}) [] = (!\text{false}, [], 0)$$

$$\mathcal{T}_p(\mathcal{E}[p\ n]) [] = (?n, \mathcal{E}, 0)$$

$$\mathcal{T}_p(\mathcal{E}[p\ n])(b :: bs) \simeq \mathcal{T}_p(\mathcal{E}[\mathbf{return\ } b])(bs)$$

$$\text{If } M \rightsquigarrow N \text{ then } \mathcal{T}_p(M)(bs) \simeq \mathcal{I}(\mathcal{T}_p(N)(bs))$$

$$\text{where } \mathcal{I}(\ell, \mathcal{E}, i) = (\ell, \mathcal{E}, i + 1)$$

Define $\text{Model} \doteq \text{Comp} \rightarrow (\mathbb{B}^* \times \text{Eval} \times \mathbb{N})$.

Standard decision trees

We are interested in predicates whose models are complete binary trees, and query each component of a provided point exactly once.

Definition (n -standard trees)

For any $n > 0$ a decision tree t is said to be n -standard whenever

- The domain of t consists of all the lists whose length is at most n , i.e.

$$\text{dom}(t) = \{bs : \mathbb{B}^* \mid |bs| \leq n\}$$

- Every leaf node in t is an answer node, i.e. for all $bs \in \text{dom}(t)$

$$\text{if } t_\ell(bs) = !b \text{ then } |bs| = n$$

- There are no repeated queries in t , i.e. for all $bs, bs' \in \text{dom}(t), j \in \mathbb{N}$

$$\text{if } bs \sqsubseteq bs' \text{ and } t_\ell(bs) = t_\ell(bs') = ?j \text{ then } bs = bs'$$

where $bs \sqsubseteq bs'$ means bs is a prefix of bs' .

Theorem

- 1 For every n -standard predicate $pred$, the generic search procedure has at most time complexity

$$\text{Time}(\text{genericSearch } pred) = \sum_{bs \in \mathbb{B}^*, |bs| \leq n} t_s(bs) + \mathcal{O}(2^n)$$

- 2 Every generic counting function $count \in \mathcal{L}$ has for every n -standard predicate $pred$ at least time complexity

$$\text{Time}(count \ pred) = \sum_{bs \in \mathbb{B}^*, |bs| \leq n} 2^{n-|bs|} t_s(bs) + \mathcal{O}(n2^n)$$

Proving the positive result

Define suitable evaluation state computing functions

$$start, end : \mathbb{B}^* \times \text{Model} \rightarrow \text{Comp}$$

Lemma

Suppose t is a model of a n -standard predicate, then for every boolean list $bs \in \mathbb{B}^*$

$$\begin{aligned} & start(bs, t) \\ \longrightarrow^+ & start(\text{true} :: bs, t) \longrightarrow \sum_{|bs|+1 \leq n} t_s(\text{true} :: bs) + 2^{n-(|bs|+1)} end(\text{true} :: bs, t) \\ \longrightarrow^+ & start(\text{false} :: bs, t) \longrightarrow \sum_{|bs|+1 \leq n} t_s(\text{false} :: bs) + 2^{n-(|bs|+1)} end(\text{false} :: bs, t) \\ \longrightarrow^+ & end(bs, t) \end{aligned}$$

Proof.

Proof by downward induction on the list of booleans bs . □

Proving the negative result

Suppose that we have an arbitrary implementation of generic search $count \in \mathcal{L}$. Pick any n -standard predicate $pred$ and look at the computation arising from $count\ pred$. Now we need to show that

Lemma (Every leaf is visited (A))

The computation ($count\ pred$) visits every leaf in the model of $pred$.

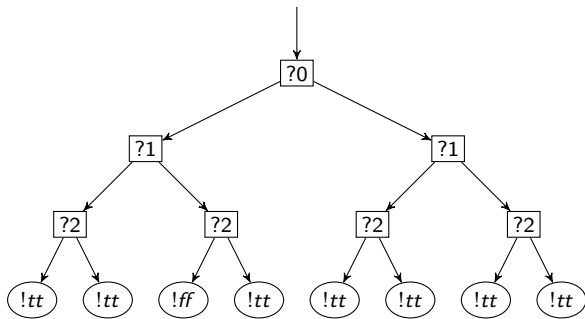
Lemma (No shared computation (B))

If p and p' are distinct points then their subcomputations are disjoint.

Since each subcomputation has length at least $\Omega(n)$ the entire computation must have at least length $\Omega(n2^n)$.

Threads and sections

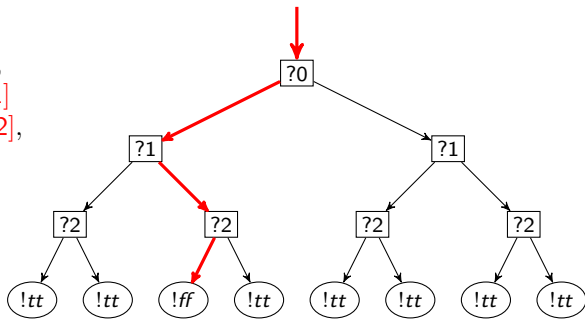
Consider a 3-standard predicate *seven* (has seven true leaves)



Threads and sections

Consider a 3-standard predicate *seven* (has seven true leaves)

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 $\mathcal{E}_0[\text{true}] \rightsquigarrow^* \mathcal{E}_1[p\ 1]$
 $\mathcal{E}_1[\text{false}] \rightsquigarrow^* \mathcal{E}_2[p\ 2],$
 $\mathcal{E}_2[\text{true}] \rightarrow \text{false} \}$

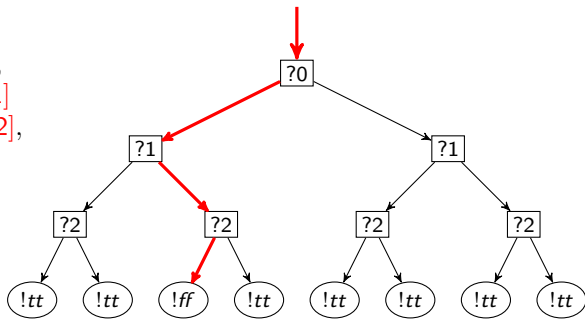


Any n -standard predicate has 2^n threads, and every thread consists of $n + 1$ sections.

Threads and sections

Consider a 3-standard predicate *seven* (has seven true leaves)

Thread $\doteq \{$ $pred\ p \rightsquigarrow^* \mathcal{E}_0[p\ 0]$,
 $\mathcal{E}_0[true] \rightsquigarrow^* \mathcal{E}_1[p\ 1]$
 $\mathcal{E}_1[false] \rightsquigarrow^* \mathcal{E}_2[p\ 2]$,
 $\mathcal{E}_2[true] \rightarrow false \}$



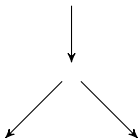
Any n -standard predicate has 2^n threads, and every thread consists of $n + 1$ sections.

Proof of Lemma A.

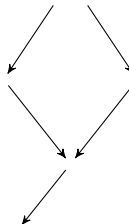
By contradiction: pick a leaf that has no thread; negate the value at the leaf; tweak the predicate accordingly; observe a wrong result. □

No shared computation

Every section has a unique successor



Every section has a single predecessor



Proof.

Follows by definition of section and the semantics being deterministic.

Proof.

By direct calculation on the reduction sequence induced by a section.

Summary and future work

In summary

- We have defined two languages \mathcal{L} and \mathcal{L}_{eff}
- We have demonstrated that \mathcal{L}_{eff} provides strictly more efficient implementations of generic search than \mathcal{L} ($\mathcal{O}(2^n)$ vs $\Omega(n2^n)$)
- ... which establish a new complexity result for control operators

Future considerations

- Perform empirical experiments to observe the result in practice (Daniels 2016)
- Study the robustness of the result, i.e. what feature(s) can we add to \mathcal{L} whilst retaining an efficiency gap between \mathcal{L} and \mathcal{L}_{eff} ?
- Generalise the result to all conceivable effective models of computations

References I

- Bauer, Andrej (2011). *How make the "impossible" functionals run even faster*. Mathematics, Algorithms and Proofs, Leiden, the Netherlands. url: <http://math.andrej.com/2011/12/06/how-to-make-the-impossible-functionals-run-even-faster/>.
- Berger, Ulrich (1990). "Totale Objekte und Mengen in der Bereichstheorie". PhD thesis. Munich: Ludwig Maximilians-Universität.
- Daniels, Robbie (2016). "Efficient Generic Searches and Programming Language Expressivity". MA thesis. Scotland: School of Informatics, the University of Edinburgh.
- Escardó, Martín Hötzel (2007). "Infinite sets that admit fast exhaustive search". In: *LICS*. IEEE Computer Society, pp. 443–452.
- Hillerström, Daniel and Sam Lindley (2016). "Liberating effects with rows and handlers". In: *TyDe@ICFP*. ACM, pp. 15–27.
- Hillerström, Daniel et al. (2017). "Continuation Passing Style for Effect Handlers". In: *FSCD*. Vol. 84. LIPIcs. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 18:1–18:19.

- Levy, Paul Blain, John Power, and Hayo Thielecke (2003). “Modelling environments in call-by-value programming languages”. In: *Inf. Comput.* 185.2, pp. 182–210.
- Longley, John (2009). “Some Programming Languages Suggested by Game Models (Extended Abstract)”. In: *Electr. Notes Theor. Comput. Sci.* 249, pp. 117–134.
- Longley, John and Dag Normann (2015). *Higher-Order Computability. Theory and Applications of Computability*. Springer.